EXPERIENTIA

Vol. XX - Fasc. 11 Pag. 593-656 15. XI. 1964

# The Problem of Continuity or Discontinuity in the Fundamental Concepts of Theoretical Physics

By R. FÜRTH

formerly of Birkbeck College, University of London (England)*

The problem as to whether matter can be subdivided into arbitrarily small parts or whether this process has a natural limit did already occupy the minds of the early Greek philosophers almost two and a half millenia ago[1]. Although these men may be regarded as the earliest 'natural philosophers' they were not really scientists in the now accepted sense of this term because they were of the opinion that problems of this kind and similar ones could be solved by mere speculation. On account of our better understanding of the nature of scientific research we can thus understand why different philosophical schools did arrive at different conclusions about these problems.

The 'Eleatic school' considered it as self-evident that there was no limit to the process of subdividing material bodies into smaller and smaller parts, and that therefore matter had the property of 'continuity'. The chief representative of this school, ZENO of Elea, was well aware of the conceptual difficulties of the notion of continuity which he formulated in the form of a number of 'paradoxa'. In particular he argued that a piece of matter consisting of infinitely small parts must itself consequently be infinitely small; but at the same time, as it consisted of an infinitely large number of parts, it ought to be infinitely large itself. He did not realize that the product of an infinitely small and an infinitely large 'number' could be finite, a mathematical notion which is indeed very difficult to understand.

The 'atomistic school', which was founded by LEUKIPPOS and DEMOKRITOS of Abdera, on the other hand, maintained that the concept of an unlimited divisibility of matter was absurd and that in fact matter consisted of the smallest particles, the 'atoma' (the indivisible), which could not be broken up any further and were separated from each other by 'empty space'.

In the present paper I shall attempt to analyse these problems from the point of view of modern physics, but first I want to say a few words on the mathematical formulation of continuity. The condition for a function $f(x)$ of one variable quantity $x$ to be continuous is that the function must be *defined* for all values of $x$ and that the derivative $df/dx$, that is the limiting value of $\Delta f/\Delta x$ for arbitrarily small values of $\Delta x$, must exist everywhere. Similarly the condition for a function of several variables, e.g. the co-ordinates $x$, $y$, $z$ of a point in space, to be continuous is that the function must be defined for all values of $x$, $y$, $z$ and that the partial derivatives $\delta f/\delta x$, $\delta f/\delta y$, $\delta f/\delta z$ must exist everywhere. One of the variables may also denote time $t$, and a function $f(t)$ of time is then called continuous if it is defined for all values of $t$ and if the derivative $\delta f/\delta t$ exists everywhere.

What then is the meaning of the statement that matter fills space continuously? The physical quantities which describe the distribution of matter in space, like mass density, electric charge density etc. are defined by the following process of measurement. One measures the total mass $\Delta M$ or the total electric charge $\Delta Q$ in a small volume $\Delta V$ and calculates the quotients $\Delta M/\Delta V$ and $\Delta Q/\Delta V$. One now repeats this procedure on smaller volumes $\Delta V$ and tries to continue further with ever decreasing volume elements $\Delta V$. *In practice* one will, of course, always eventually reach a limit to this process imposed by the finite sensitivity of the measuring instruments.

Now the continuum theory supposes that nevertheless *in principle* the process can be continued indefinitely and that the limiting values

$$\varrho = \lim_{\Delta V \to 0} \frac{\Delta M}{\Delta V} = \frac{dM}{dV} \quad \text{and} \quad \sigma = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$

exist and are continuous functions of space in the above defined mathematical sense of the word. The discontinuum theory, on the other hand, assumes that the described measuring process *cannot* be continued indefinitely, and that the physical quantities defined by this procedure of measurement are discontinuous functions of space. This is essentially equivalent to the assumption that matter consists of indivisible elementary particles and that, for example, the mass density within such a particle has a definite

* Substance of a lecture given to the 'Studium Generale' of the University of Mainz in May 1964.

[1] P. DEUSSEN, *Allgemeine Geschichte der Philosophie* (Leipzig 1914), vol. II.

finite value, equal to the mass of the particle divided by its volume, while outside the particles the mass density is zero. We shall replace this rather superficial consideration by a more thorough analysis of the measuring process later on.

The history of physics shows that at various stages in its development it was alternately dominated by the notions of continuity and discontinuity. But at the beginning of this century, when what is now called classical physics' had reached the summit of its perfection, it was believed that it was necessary to use discontinuity and continuity notions together in order to explain the then known physical phenomena. It was assumed that matter and electricity were built up discontinuously of smallest particles, but that apart from these there existed the continuous gravitational and electromagnetic fields, consisting in some sort of modifications in the immaterial continuous 'aether'. This, however, did not necessarily mean that mass and electric density had to be regarded as discontinuous functions of space in the strict mathematical sense; for one could imagine the elementary particles to be simply 'singularities' of the continuous fields within which the density functions were still continuous but had infinities in the centres of the particles.

The physical quantities are, of course, in general not only functions of space but also functions of time. The immediate observation seems to show that the movement of a body through space must be a continuous process, that is, that a 'mass point' cannot get from one point of its path to another without passing through all intermediate points. Quite generally classical physics may be characterized by three fundamental principles, namely the principle of the continuity of all physical processes in time, the principle of causation, and the principle of the existence of a preferential system of reference for the mathematical description of the laws of physics, essentially identical with the 'aether'. But it must be realized that the concept of a continuous motion is by no means simple. Indeed the previously mentioned Greek philosopher ZENO[1] found it impossible to understand how an arrow could move at all when it must be supposed to occupy a definite position in space at any moment and therefore seemingly to be at rest at every moment.

If one tries to solve this problem by way of experiment, for example by making a cinematographic record of a moving object, one will find that, provided the speed of the recording camera is great enough, on each single 'frame' of the film the object indeed seems to be at rest. Modern physics has abandoned the continuity axiom altogether, like the two other axioms of classical physics, and thus resolved this difficulty radically. But in order to understand properly what is actually meant by continuity and discontinuity in time of physical processes we shall have to give a more detailed analysis of the measuring process itself.

It is clear that the determination of the numerical value of a physical quantity is only possible by means of a suitable measurement, and if these numerical values are to have a definite meaning the definition of a physical quantity must always contain a statement about the way in which the measurement is to be carried out; this is called an 'operational' definition[2]. In order to perform a measurement one needs a measuring instrument (the human sense organs also falling into this category), and this instrument must be made to interact with the object to be measured as otherwise a measurement is clearly impossible. It has also to be taken into account that every measuring device has only a finite 'resolving power' and that it must have a certain degree of inertia.

On account of the finite resolving power of all measuring instruments it appears that in the process of the examination of a spatial structure by means of a suitable recording device, e.g. a microphotometer, there must exist a finite spatial interval $\Delta s_0$ such that the corresponding change $\Delta f_0$ of the physical quantity to be measured is just detectable. The difference quotient $\Delta f_0/\Delta s_0$ therefore here replaces the differential quotient $df/ds$ and will evidently always be finite. Hence the recorded function $f(s)$, which represents the spatial structure, must necessarily be 'macroscopically continuous', and strictly speaking one can never decide whether the structure is 'actually' continuous or discontinuous.

Supposing now that with increasing magnification and increasing resolving power of the recording instrument the diagram appears to become smoother and smoother, one will then be inclined to conclude that the structure in question is indeed continuous although this cannot be verified. On the other hand it may happen that the diagram becomes more and more complicated and 'spiky' the higher the magnification; one will then have reason to believe that the structure is really discontinuous. But it may also occur that the character of the picture repeatedly changes while the magnification is increased. As an example consider the distribution of 'density' in a photographic plate as used for producing a printing block for a newspaper picture. Viewed by the naked eye the density distribution seems to be continuous, but observation through a magnifying glass shows that the pattern consists of isolated dark patches. With improved magnification one will find that within each of these patches there is again a continuous density distribution; but if the plate is observed through a powerful microscope one sees that in fact the apparently continuous optical density has resolved itself into isolated opaque silver grains between which the plate is transparent.

[2] The concept of 'operational definition' is due to P. W. BRIDGMAN, *The Logic of Modern Physics* (New York 1949).

Similar considerations may be applied to the observation of the course of a physical process in time, again by means of some recording device. Because, as mentioned before, all such instruments have a certain amount of inertia and will therefore 'integrate' over a small but always finite interval $\Delta t_0$ of time, usually called its 'relaxation time', the difference quotient $\Delta f_0/\Delta t_0$ again stands for the ideal differential quotient $df/dt$. $\Delta f_0/\Delta t_0$ is a macroscopically continuous function of time, and it follows that the recorded diagram will always appear to be continuous and differentiable everywhere.

If the recording of the same process is repeated by means of an instrument with higher sensitivity and shorter relaxation time the appearance of the diagram will change and the curve will either become more and more regular or more and more irregular. In the former case we shall be justified in concluding that the process is continuous in time, in the latter that it takes place in the form of discontinuous 'jumps'. But here again it may happen that the character of the record changes periodically while the sensitivity of the recording device is increased. As an example we choose the observation by means of a microscope of the 'Brownian movement' of a small particle, suspended in a liquid[3]. The curve, representing the displacement of the particle in successive time intervals, becomes more and more irregular the higher the magnification of the microscope and the shorter the time intervals between observations. From this, one would be tempted to conclude that the velocity of the Brownian movement of the particle is really a discontinuous function of time. But if one could improve the method of observation to a point where it would be possible to follow the movement of the particle between successive impacts with the molecules of the surrounding medium one would doubtless find that the diagram within such a time interval has become smooth again.

Another example is provided by the observation of the changes in time of the current intensity in an electric circuit by means of a cathode ray oscillograph with amplifier or a similar device possessing a very short relaxation time. If the amplification is made sufficiently large, an irregular fluctuation appears to be superposed over an otherwise smooth curve, and this irregularity becomes more and more pronounced the higher the amplification. But here it would never be possible to observe the changes in current produced by the movement of the separate electrons in the circuit because the statistical character of the observed fluctuation can be shown to be entirely determined by the macroscopic electric properties of the circuit elements.

By means of the various ingenious devices developed during the last 50 years or so, like the cloud, bubble, and spark chambers, the photographic plate with 'nuclear emulsion' and the different types of particle counters, it has become possible to count the number of particles released in the course of physical processes and to make their individual paths visible. There can therefore be no doubt that matter is indeed built up of elementary particles. Nevertheless it has become clear from fundamental theoretical considerations that it is not possible to follow the movement of these particles in detail, that is to determine their positions in space as functions of time in the sense of classical physics.

As already mentioned, the quantum theory has had to abandon the classical principle of continuity for the changes in time of the physical quantities, and it maintains that it is impossible on principle grounds to perform measurements for verifying this principle. For, according to this theory, the changes in the state of physical systems occur discontinuously in the form of 'quantum jumps', as for example the changes in the state of single atoms, in which the energy of the atoms changes abruptly. Consequently the interaction between different atoms must take place in the form of exchanges of *finite* amounts of energy. This fundamental aspect of physical interaction processes is reflected in the appearance of the universal 'Planck constant' $h$ in the basic equations of the quantum theory.

This fact has an important consequence for the measuring process. For, according to what has been said before, these processes are necessarily exchange processes between the object to be measured and the measuring instrument and can therefore not be refined indefinitely as was assumed in classical physics. Quantitatively this is expressed by the so-called 'Heisenberg uncertainty principle' (first formulated in 1927), according to which the combined accuracy of the simultaneous measurements of position and momentum of a particle is of the order of magnitude $h$[4]. When one therefore tries to follow up as precisely as possible the changes in time of a physical quantity one will find that the function $f(t)$, representing these changes, does not become smoother with increasing refinement of the means of observation but on the contrary seems to become more and more irregular. This means that either the function $f(t)$ is discontinuous or that it is not differentiable. An example is provided by the fluctuations which become apparent in the observation of atomic phenomenon like radioactive disintegration processes.

When in particular one attempts to measure in detail the movement of an elementary particle one observes that its trace in say a photographic emulsion has always a slightly 'wobbly' appearance which is

[3] For a discussion of this and similar problems see R. FÜRTH, Sci. Progr. *143*, 396 (1948); Sci. Am. *183*, 48 (1950).

[4] W. HEISENBERG, *The Physical Principles of Quantum Theory* (Cambridge 1930).

due to the collisions of the particle with the molecules of the emulsion by which the trace is produced. One therefore sees that, strictly speaking, the velocity of an elementary particle as the derivative of its position function with respect to time cannot be operationally defined. It could, of course, be maintained that a particle, which does not interact with its surroundings at all, moves with constant velocity as demanded by the classical law of inertia. But from what has been just said it is clear that it would be impossible to verify this statement, so that, in terms of modern philosophy of science, it would be 'meaningless'.

A much more satisfactory way of interpreting these observations is to say that the elementary particles indeed perform a random zig-zag or 'wobbling' motion which is superposed over the classical regular motion, as formulated by SCHRÖDINGER[5]. This movement shows a certain formal similarity to the previously mentioned Brownian movement of 'macroscopic' colloidal particles, and it can indeed be shown that the latter imposes a limit to the accuracy of macroscopic measurement which can be formulated by an 'uncertainty principle' that is formally analogous to the quantum-mechanical Heisenberg principle[6]. But whereas the wobbling motion of the elementary particles persists at zero temperature and the order of magnitude of the quantum-mechanical uncertainty is given by the universal constant $h$, the Brownian motion result from the irregular thermal movement of the molecules and therefore vanishes at the zero point of temperature $T$, and the relevant uncertainty is of the order $kT$.

In most cases it is, of course, not possible at all to follow the movement of the individual elementary particles by means of macroscopic devices. For example, one cannot observe the movement of individual electrons in the interior of material bodies; but it is possible to determine the relative duration of time the electrons spend in the various volume elements of the body from diagrams of the diffraction of X-rays by the body. The electron density function calculated in this way corresponds roughly to the distribution of the density in a time exposure photograph of a multitude of fast moving entities.

Are we then, according to what has been said so far, justified in maintaining that the physical phenomena are essentially discontinuous in space and time? This remains doubtful. For what one can say with certainty is only that the classical fundamental laws of mechanics and electrodynamics, which are based on the continuum aspect, cannot be correct. But the fundamental equations of quantum theory (quantum mechanics and quantum electrodynamics) are also differential equations, though not for the directly observable physical parameters but for the so-called 'wave functions' from which the expectation values of these parameters can be calculated[7]. This means that we suppose the wave functions to be continuous functions in the strict mathematical sense which can be differentiated with respect to the co-ordinates and time.

Furthermore, advanced experimental technique has in recent years made it possible to obtain information about the internal structure of the elementary particles themselves, although at a certain stage in the development of quantum theory it was believed that the very notion of 'internal structure' of elementary particles was meaningless. As already indicated before, one is able to determine electronic distribution functions experimentally, for instance to determine the distribution of electrons in the outer shell of an atom by a kind of photographic time exposure. What is revealed in this way is a continuous 'smeared out' electric density distribution in an electronic cloud surrounding the atomic nucleus, which may justifiably be considered to represent the atomic structure.

One can proceed in a similar way if one wants to determine the structures of the 'heavy' elementary particles, the proton and the neutron. But one then has, instead of X-rays, to use very energetic beams of electrons or 'mesons' and observe in what way these particles are scattered when they pass 'through' a proton or a neutron. A great deal of work has in recent years been done in this field of high energy research, and it has been possible to establish that the heavy elementary particles indeed possess a definite internal spatial structure. This fits harmonically into the mentioned fundamental notions of quantum mechanics concerning the continuous character of the wave function. For according to quantum theory a material particle is equivalent to a packet of 'de Broglie waves'. Thus we see that in a certain sense we have returned to the classical concept of the elementary particles as 'nodes' or singularities in a continuous field.

Does this now mean that modern physics has after all in fact adopted the continuum aspect of physical phenomena? On the contrary, for the situation can be considered from a completely different point of view. It was already pointed out that from the point of view of quantum mechanics a material particle is equivalent to a continuous field of de Broglie waves. *Vice versa*, however, it is also known from various experiments, e.g. the 'photoelectric effect' and the 'Compton effect', that the energy of an electromagnetic wave is not spread out continuously in space but is concentrated in the light quanta or 'photons'[8]. This means that a

[5] E. SCHRÖDINGER, Berl. Ber. *1930*, 296.

[6] R. FÜRTH, Z. Phys. *81*, 143 (1933).

[7] The statistical interpretation of the quantum-mechanical wave function was first proposed by M. BORN in 1926; see e.g. M. BORN, *Natural Philosophy of Cause and Chance* (Oxford 1948) and *Physics in my Generation* (London 1956).

[8] The concept of photons was introduced into the quantum theory by A. EINSTEIN in 1905.

particular type of non-permanent particles must be associated with a continuous electromagnetic wave field.

Beyond this quantum electrodynamics maintains that *all* electromagnetic interaction phenomena may be explained by processes of exchange of energy and momentum between systems by means of photons. In order to understand this better we have to remember that all macroscopic mechanical forces, like the pressure in a gas or the elastic stresses in a solid, which seemingly obey phenomenologically continuous laws, do in fact consist of exchanges of energy and momentum between the molecules of material bodies.

An analogous situation exists in the theory of the 'nuclear forces', that is the forces which act between the 'nucleons', the constituents of the atomic nuclei, and which hold them together. These forces differ basically from the electromagnetic and the gravitational forces in so far as they act only over very short distances of the order of magnitude of the nuclear dimensions. It was first suggested by YUKAWA (1935), that a new type of particle should be assigned to the continuous field of the nuclear forces which, just as the photons associated with electromagnetic fields, had no permanence and could be created and annihilated. On the basis of quantum theoretical considerations he came to the conclusion that these particles, in contrast to the photons, should have a finite 'rest mass' lying between the masses of the 'light' elementary particles (leptons) and the heavy particles (baryons); they are therefore called 'mesons'.

These ideas have been fully confirmed by experiment, and a whole series of different types of mesons are now known, whose paths can be made visible by the previously mentioned devices and whose properties can be examined in detail in this way. They were first detected in the naturally occurring cosmic radiation, but are nowadays mainly studied in the high energy laboratories where they can be created by means of gigantic particle accelerators and where their interaction processes can be investigated. The so-called '$\pi$-mesons' in particular are thought to be mainly responsible for the nuclear forces, and one imagines that, for example, the field of force surrounding a proton consists in the proton's emitting and almost immediately re-absorbing these mesons, like a juggler throwing and catching balls. The interaction between two nucleons then takes place through the exchange of $\pi$-mesons between them.

If one develops these ideas consistently one is led to the conclusion that *all* physical phenomena may be due to the interaction of permanent and non-permanent elementary particles which have the character of direct 'collisions'. The various ways in which these collisions occur can be studied in detail by means of the mentioned devices, but nothing can be found out about the behaviour of the particles *between* the collisions; for, according to what was said before, all observations consist of interactions between object and measuring instrument and therefore cannot be made without collisions with the observed entity taking place.

Following this trend of ideas one may also connect the classical continuous gravitational field with a special kind of particle, and in recent attempts to formulate a quantum theory of gravitation these particles, called 'gravitons', play an essential part. So far, however, it has not been possible to verify these hypotheses experimentally and to observe gravitons directly. Incidentally, the idea of explaining the gravitational attraction of material bodies by the impacts on them of a peculiar type of 'aether particles' is quite old but has never been taken seriously.

The modern theory of solids has further led to the notion of the so-called 'phonons'. It was mentioned before that the quantum theory assigns a photon to a plane monochromatic electromagnetic wave; similarly one may imagine a mechanical or 'acoustic' wave of the same description to be equivalent to a hypothetical particle which is therefore called a phonon. Now since the molecular movement in a material body can be resolved into a system of waves with a quasi-continuous spectrum, one imagines such a body to be filled with a 'phonon gas' just as the electromagnetic radiation contained in an enclosure is considered to be equivalent to a 'photon gas'.

The notion of phonons has proved itself to be an extremely useful heuristic hypothesis for the development of the theory of thermal and electric phenomena in solids. In my opinion, however, one should beware of ascribing to the phonons and similar recently 'invented' quasi-particles the same sort of reality as the photons. For the classical electromagnetic field is a true continuum while a mechanical or acoustic wave is nothing but a co-ordinated movement of individual molecules or atoms and therefore constitutes a typical discontinuum. Hence the energy and momentum of such a wave are surely the sums of the energies and momenta of the separate molecules and cannot be concentrated in small volumes of space in the form of special particles.

It appears that there is, at the present time, a tendency in theoretical physics to consider the physical world as a discontinuous interaction play in time between discontinuous elementary particles in space. If one takes this point of view the question arises whether under these circumstances one is at all justified in retaining the classical continuum notions of space and time and whether one should not endeavour to base the mathematical description of physical phenomena on the concept of a universal fundamental length and a universal fundamental time, so that all lengths and times would be integer multiples of these fundamental units. In this way one would also avoid altogether the numerous mathematical difficulties which are inherent in the concept of a continuum.

The idea of introducing a fundamental length as a basic element into the theory of the elementary particles has been proposed independently by a number of scientists[9], and I myself attempted 35 years ago to develop a 'theory of elementary uncertainties' which essentially rests on this concept[10]. If one assumes this fundamental length to be a fraction of the nuclear dimensions, say $10^{-14}$ cm, then the fundamental time ought to be of the order of the fundamental length divided by the velocity of light, e.g. about $10^{-24}$ sec. This would explain why the notion of wave functions, continuous in time and space, has worked so satisfactorily in the application of quantum theory to the atom and why this theory, at least in its present continuous form, does not seem to be capable of solving the problem of the elementary particles.

It is worth mentioning that PLANCK, the founder of quantum theory, clearly pointed out that the existence of the 'quantum of action' $h$ was equivalent to a cell structure with 'cells' of volume $h^n$ to the $2n$-dimensional 'phase space' of a system of $n$ degrees of freedom[11]. The concept of a fundamental length and a fundamental time constitutes an even more radical aspect of discontinuity, namely the notion of a cell structure of the classical space-time continuum itself.

Will the future development of fundamental physical theory proceed in this direction, and can one expect this to lead to the solution of the problem of the elementary particles? Or has, on the contrary, a radical continuum theory a better prospect? These questions cannot be answered at present, and it is indeed possible that they cannot be answered at all. According to BOHR[12], for example, the concepts of discontinuity and continuity (or particle and wave) are not contradictory but 'complementary' to each other; that is to say, that for a full description of a physical phenomenon both aspects have to be taken into account, and that it depends on the experimental situation which of the two aspects is more appropriate.

As all knowledge of the physical world is obtained by the interplay of the analytical process of the interpretation of experimental results and free speculation, it is not altogether surprising that the laws of nature may be formulated in different ways and that, as a result of the limitations of the human mind, it may indeed be impossible to formulate these laws in a consistent manner. So it is quite possible that the question posed by this article cannot be answered at all and that none of the extreme concepts of the physical world can exist without the other.

*Zusammenfassung*. Das Problem der Kontinuität oder der Diskontinuität der räumlichen Struktur der Materie und des zeitlichen Ablaufs der physikalischen Vorgänge wird vom Standpunkt der modernen theoretischen Physik kritisch beleuchtet. Es wird zunächst diskutiert, inwieweit man diese Frage überhaupt auf Grund von experimentellen Beobachtungen entscheiden kann. Es werden dann die experimentellen Beobachtungen aufgezählt, aus denen hervorzugehen scheint, dass die Materie den Raum diskontinuierlich in der Form von Elementarteilchen erfüllt und die physikalischen Elementarvorgänge diskontinuierlich in der Zeit vor sich gehen. Dem steht entgegen, dass es neuerdings gelungen ist, eine innere Struktur der Elementarteilchen festzustellen, und dass die Grundgleichungen der Quantentheorie die Form von Differentialgleichungen für kontinuierliche «Wellenfunktionen» haben. Dies bedeutet jedoch nicht, dass die moderne Physik doch wieder im Prinzip den Kontinuumstandpunkt eingenommen hat. Im Gegenteil besteht derzeit die Tendenz, alle physikalischen Wechselwirkungsvorgänge auf den zeitlich diskontinuierlichen Austausch von unbeständigen, neuartigen Elementarteilchen zwischen den räumlich diskontinuierlichen Elementarbausteinen der Materie anzusehen. Es erhebt sich die Frage, ob nicht vielleicht auch die Kontinuumvorstellung von Raum und Zeit selbst aufgegeben werden sollte. Zum Schluss wird kurz darauf hingewiesen, dass man möglicherweise die physikalische Welt nur begreifen kann, wenn man sowohl Kontinuum- als auch Diskontinuumvorstellungen gleichzeitig benützt.

[9] A. MARCH, *Natur und Erkenntnis* (Wien 1948).
[10] R. FÜRTH, Z. Phys. *57*, 429 (1929).
[11] M. PLANCK, Ann. Phys. *50*, 385 (1916); *Physikalische Abhandlungen und Vorträge* (Braunschweig 1958), vol. II.
[12] N. BOHR, *Atomic Physics and Human Knowledge* (New York 1958).